COMMENTS TO
G. GRUBB: “DISTRIBUTIONS AND OPERATORS”

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Corrections, *updated January 15, 2011*.

Notation: x means page x, with x^y indicating line y from above, x_y indicating line y from below.

4^{15} replace “differention” by “differentiation”

13^{19+23} replace “$p_{j,k}$” by “$p_{k,j}$”

18^{12+18} replace “L^1” by “L_1”

24_{8+7} replace “$i=0$” by “$j=0$”

24_7 add the sentence “The conclusion of Theorem 2.17 also holds when the V_j are arbitrary open sets, since they can be replaced by bounded sets $V_j \cap B(0,R)$ with R taken so large that $K \subset B(0,R)$.”

42^5 replace “$\check{(}\varphi$” by “$(\check{\varphi}$”

42_{10} replace “(2.35)” by “(2.32)”

44_{15} replace “$J \circ T^{-1}$” by “JT^{-1}”

60^{13} add the line “here $\partial f = g$.”

62_{14} replace “(C.11)” by “(C.10)”

63^4 replace “v” by “u” in two places

64_7 replace “$\chi_N u$” by “$\chi_N u = \chi(x/N)u$”

65^{2+5+6} replace “L^2” by “L_2” in the subscripts

65_2 replace “$B(0,\frac{1}{j})$” by “$B(x,\frac{1}{j})$”

66 let the footnote refer to (3.60) instead of (3.43)

72_{11} change the definition of $\tilde{v}_\delta$ to

$$\tilde{v}_\delta(x) = \tilde{u}(\tfrac{\alpha+\beta}{2} + \tfrac{1}{1-\delta}(x - \tfrac{\alpha+\beta}{2}))$$

73^9 replace “perioodic” by “periodic”

76^{16} replace “$m-1)$” by “$m-1$”

79_9 replace “$dy_n dx'$” by “$dx' dy_n$”

83_9 replace “$\Omega_b = \{x \in \mathbb{R}^n \mid 0 \le x_j \le b\}$” by “$\overline{\Omega}_b$, where $\Omega_b = \{x \in \mathbb{R}^n \mid 0 < x_j < b\}$”

84^6 replace “Ω_R” by “Ω_b”

84^9 replace “the lemma” by “Theorem 4.29”

89^7 replace “$(H, V, l(u,v))$” by “$(H, V, l_0(u,v))$”

126_{14} replace “Exercise 12.36” by “Exercise 12.35”

126_{10} replace “this theorem” by “Theorem 6.3”

127_1 add "(The constant 4/3 can e.g. be found as the maximum of $(1+2s+2t)/(1+s+t+st)$ for $s=|x|^2$, $t=|y|^2 \in \overline{\mathbb{R}}_+$.)"
135_4 replace "$d\eta d\zeta$" by "$d\zeta d\eta$"
136^{17} replace "when u" by "when φ"
158^1 replace "Show that" by "Let $\operatorname{Re} b > -2$. Show that"
226_{9-7} "$\rho_{(m)}$" should be "$\varrho_{(m)}$" (such wrong fonts occur here and there in the book)
320^{15} replace "LiC^-" by "C^-"
349_3 replace "$|p(\xi)| \le C$ for $\xi \in X$" by "$|p(x)| \le C$ for $x \in \Omega$"
350_{11} replace "$\beta a x_2$" by "$\beta a(x_2$"
352_{13} replace "12.9 3°" by "12.9"
353_5 add the sentence "Moreover, H is dense in V^*; this is seen e.g. by observing that the mapping $f \mapsto \ell_f$ from H to V^* is the adjoint of the injection of V into H; here one can apply Theorem 12.7."
359_8 replace "at" by "as"
362^{11} replace "$e^i\theta$" by "$e^{i\theta}$"
368^{19+20} remove "see in particular Exercise 4.14"
370^{23} remove "rr"
434^5 replace "Exercise B.1" by "Exercise B.3"
436_1 the signs "$|$" are superfluous
437^{13} the signs "$|$" are superfluous
448^{18} replace "order m" by "order k"

Additional exercises.

Exercise 6.39. Denote by $\ell_2^N(\mathbb{N})$ the Hilbert space of complex sequences $\underline{x} = (x_k)_{k\in\mathbb{N}}$ with norm $\|\underline{x}\|_{\ell_2^N(\mathbb{N})} = \big(\sum_{k\in\mathbb{N}} |k^N x_k|^2\big)^{\frac12} < \infty$; the corresponding scalar product is $(\underline{x},\underline{y})_{\ell_2^N(\mathbb{N})} = \sum_{k\in\mathbb{N}} k^{2N} x_k \bar{y}_k$.

(a) Show that $V = \ell_2^1(\mathbb{N})$ and $H = \ell_2^0(\mathbb{N})$ satisfy the hypotheses around (12.36).

(b) Show that when V^* is considered as in Lemma 12.16, it may be identified with $\ell_2^{-1}(\mathbb{N})$.

(c) Let $a(\underline{x},\underline{y}) = (\underline{x},\underline{y})_{\ell_2^1(\mathbb{N})} + 2(\underline{x},\underline{y})_{\ell_2^0(\mathbb{N})}$, with domain V. Find the associated operator A in H defined by Definition 12.14, and check the properties resulting from Theorem 12.18.

Exercise 6.40. Let I be an interval of $\mathbb{R}$. Show, by construction, that the equation $Du = f$ has a solution $u \in \mathscr{D}'(I)$ for any $f \in \mathscr{D}'(I)$. Describe all solutions for a given f.
(*Hint:* The mapping from φ to ψ defined in the proof of Theorem 4.19 may be helpful.)

Exercise 6.41. Define the sesquilinear form a_1 by

$$a_1(u,v) = \int_0^\infty (u''\bar{v}'' + 2u'\bar{v}' + u\bar{v})\,dx, \quad u,v \in H^2(\mathbb{R}_+),$$

and let a_0 be its restriction to $H_0^2(\mathbb{R}_+)$. Let $H = L_2(\mathbb{R}_+)$, $V_1 = H^2(\mathbb{R}_+)$, $V_0 = H_0^2(\mathbb{R}_+)$.

(a) Show that the triples (H, V_0, a_0) and (H, V_1, a_1) satisfy the conditions for application of the Lax-Milgram theorem (Theorem 12.18).

(b) Denoting the hereby defined operators by A_0 resp. A_1, find how these operators act and what their domains are.

(c) Show that the operators are selfadjoint positive.

Exercise 6.42. Let $Q =]-1,1[\times]-1,1[\subset \mathbb{R}^2$, and let $u(x,y)$ be the function on $\mathbb{R}^2$ defined by

$$u(x,y) = \begin{cases} x+y & \text{for } (x,y) \in Q, \\ 0 & \text{for } (x,y) \notin Q. \end{cases}$$

(a) Find the Fourier transform of u.

(*Hint.* One can first determine the Fourier transform of the function 1_Q and then use rules of calculus.)

(b) Find the Fourier transforms of $D_x u$ and $D_y u$.

(c) Determine whether $u \in H^0(\mathbb{R}^2)$, and whether $u \in H^1(\mathbb{R}^2)$.

Exercise 6.43. Let $I =]-1,1[$, and let $\mathcal{B}$ denote the space of functions $\varphi \in C^\infty(\overline{I})$ satisfying $\varphi(0) = 0$. Show that $\mathcal{B}$ is dense in $L_2(I)$, but not in $H^1(I)$.

(*Hint.* Recall that convergence in $H^1(I)$ implies convergence in $C^0(\overline{I})$.)